Decentralized Optimization Algorithms for Large-Scale Deep Neural Network Training

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Aug 5, 2021, Zhejiang University

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- Primal-dual decentralized methods
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- An open source decentralized deep training framework: BlueFog

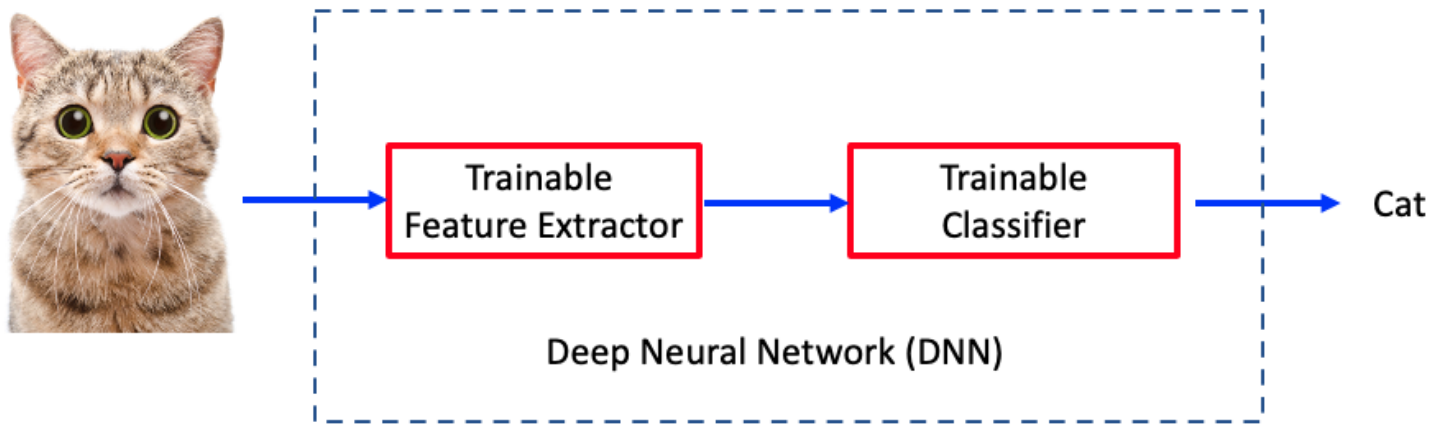
Part I: Deep Neural Network (DNN) Training Algorithms

Part I: Deep Neural Network (DNN) Training Algorithms

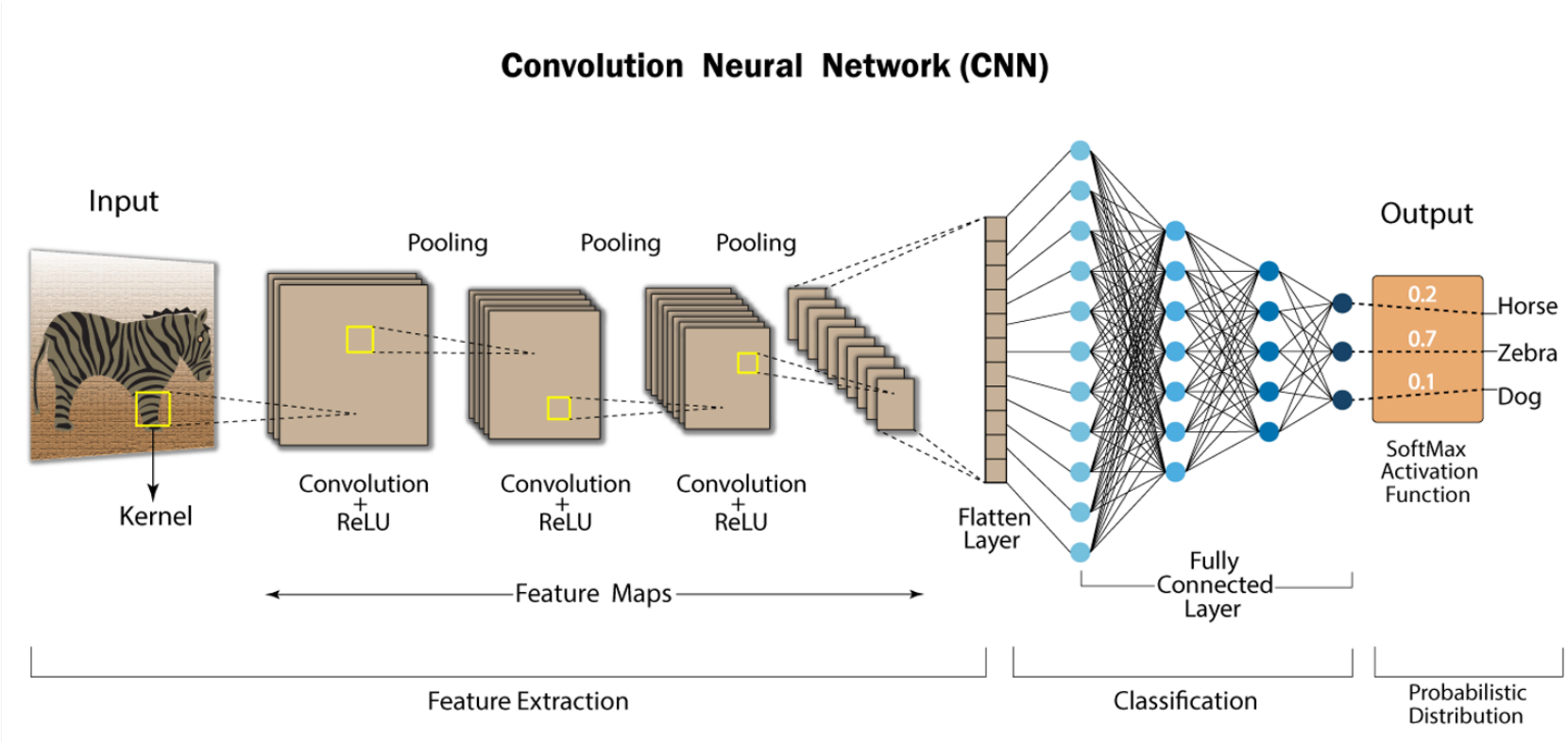
- Sec.1: Deep Neural Network Model
- Sec.2: Stochastic Gradient Descent and Single-Node Training
- Sec.3: Parallel/Distributed Training
- Sec.4: Decentralized Training

Deep Neural Network

- DNN is widely used in almost all AI applications
- A typical DNN model includes a feature extractor and a classifier
- Well-trained DNN can make precise predictions



A practical DNN example[1]

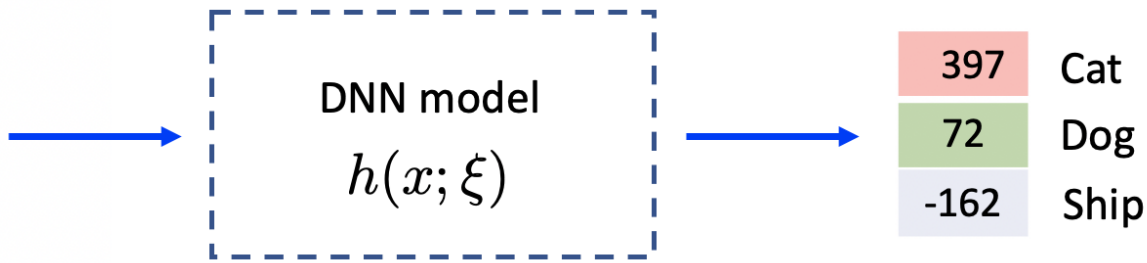


[1] Source: analyticsvidhya.com

DNN model

- We model DNN as $h(x;\xi) : \mathbb{R}^d \to \mathbb{R}^c$
  - $x \in \mathbb{R}^d$ is the DNN model parameter to be trained
  - ξ is the input data sample
  - c is the number of classes

- Given the model parameter x, DNN outputs prediction scores $\hat{y}_i$ for input ξ_i

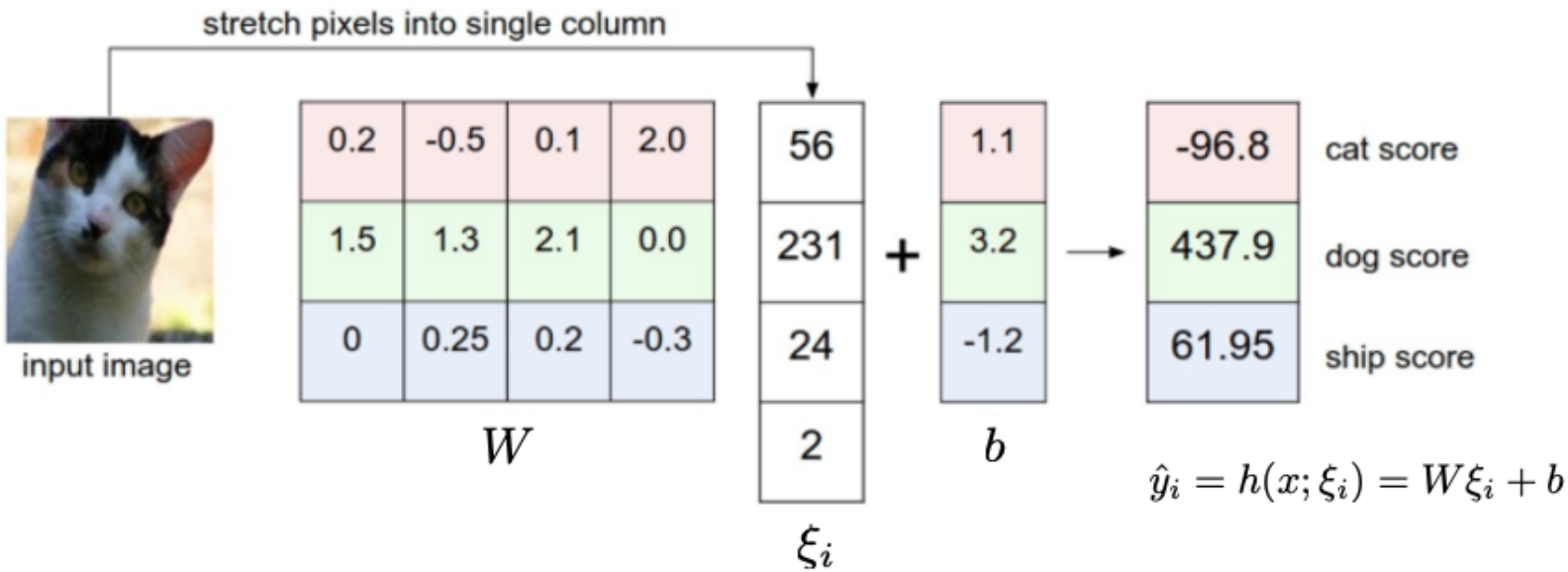


ξ_i

$\hat{y}_i = h(x;\xi_i) \in \mathbb{R}^c$

DNN model: a trivial example

- Given model parameter $x = [W; b]$, and a linear model $h(x; \xi) = W\xi + b$,
- An illustration of the trivial DNN model and its output is as follows[2]



[2] Source: https://cs231n.github.io/linear-classify/

How to train a DNN model?

- Given model parameter x, DNN $h(x;\xi)$ can make precise predictions
- But how to train/achieve the model parameter x ?
- Given a dataset $\{\xi_i, y_i\}_{i=1}^m$ where y_i is the ground-truth label for data ξ_i
- Define $L(\hat{y}_i, y_i) = L(h(x;\xi_i), y_i)$ as a loss function to measure the difference/mismatch between predictions and ground-truth labels
- DNN training is to find a model parameter x such that the mismatch (between pred and real) are minimized across the entire dataset:

$$x^\star = \arg\min_{x\in\mathbb{R}^d} \left\{ \frac{1}{m} \sum_{i=1}^{m} L(h(x;\xi_i), y_i) \right\}$$

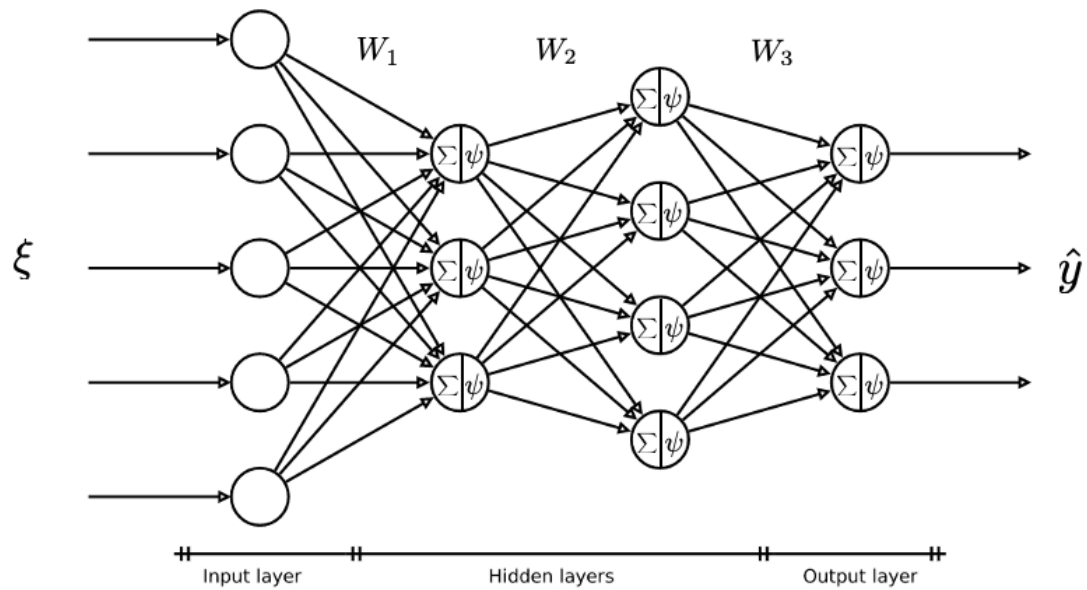
DNN model is notoriously difficult to train

- DNN model $L(h(x;\xi),y)$ is highly non-convex, and probably non-smooth

$$h(x;\xi) = \psi(\cdots\psi(W_2\cdot\psi(W_1\xi+b_1)+b_2)\cdots)$$

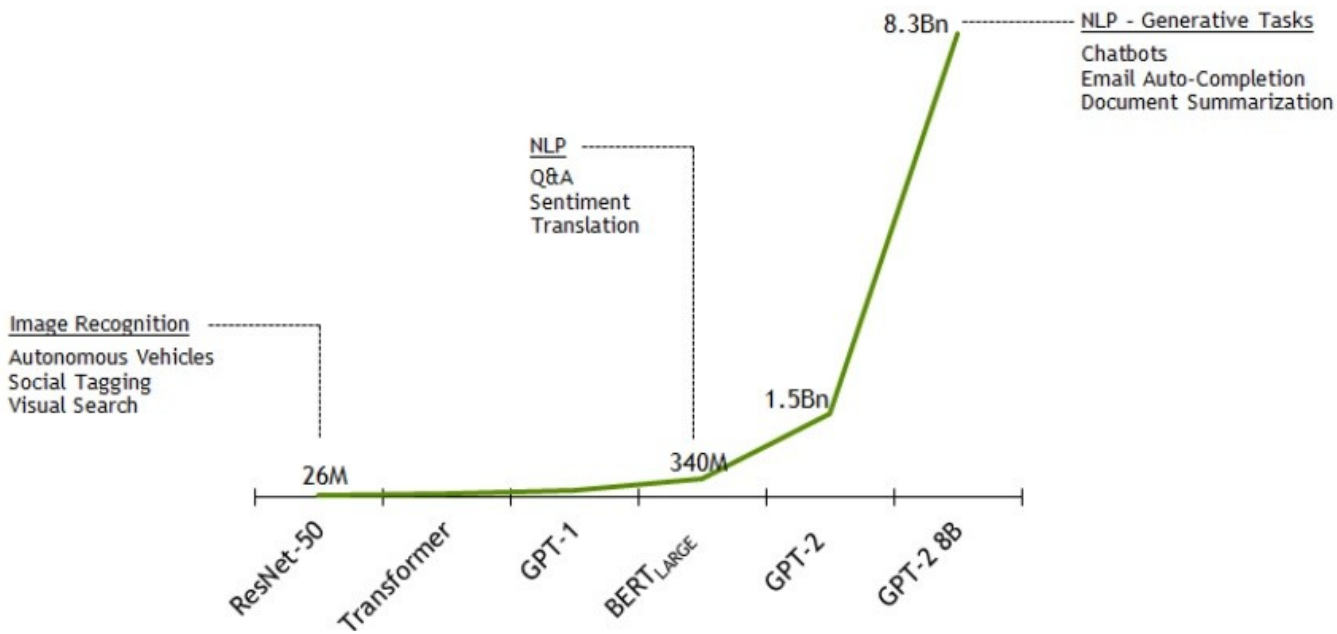
$$L(\hat{y};y) = \frac{1}{2}\|y-\hat{y}\|^2 \text{ or } -y\log(\hat{y}_i) \text{ or others}$$

where $x=\{W_i,b_i\}$ and $\psi(\cdot)$ is a non-linear activation function



DNN model is notoriously difficult to train

- Cannot find global minima; trapped into local minima and saddle points
- The dimension of model parameter $x = \{W_i, b_i\}$ (or model size) is huge[3]



[3] Image source: neowin.net

DNN model is notoriously difficult to train

- Cannot find global minima; trapped into local minima and saddle points
- The dimension of model parameter $x = \{W_j, b_j\}$ (or model size) is huge
- The size of the dataset $\{\xi_i, y_i\}_{i=1}^m$ is huge

DNN Trainig = Non-convexity training + Huge dimension + Huge dataset

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DNN model formulated as stochastic optimization

- Recall the DNN training problem

$$\min_{x\in\mathbb{R}^d} \quad \frac{1}{m}\sum_{i=1}^{m} L(h(x;\xi_i), y_i)$$

  - x is the model parameter to train; $\{\xi_i, y_i\}_{i=1}^m$ is the dataset
  - $h(x;\xi)$ is the DNN model; highly non-convex
  - $L(\hat{y}, y)$ is the loss function

- Let $\xi_i := \{\xi_i, y_i\}$ and $F(x;\xi_i) := L(h(x;\xi_i), y_i)$, the problem becomes

$$\min_{x\in\mathbb{R}^d} \quad \frac{1}{m}\sum_{i=1}^{m} F(x;\xi_i)$$

  which is a finite-sum empirical risk minimization (ERM) problem.

DNN model formulated as stochastic optimization

- When ξ follows distribution D, DNN training can also be formulated as

$$\min_{x \in \mathbb{R}^d} \; f(x) \quad \text{where} \quad f(x) = \mathbb{E}_{\xi \sim D} F(x; \xi)$$

  which is a stochastic optimization problem.

- ERM is a good approximation to the above problem, especially for large m

- In this lecture, we will focus on the above stochastic problem formulation.

Stochastic gradient descent

- D is unknown; no closed-form for $f(x)$; cannot use gradient descent
- The most popular algorithm is stochastic gradient descent (SGD) (Robbins and Monro, 1951; Bottou, 2010)
- Main idea: sample one (or one batch of) data sample and perform SGD

$$x^{(k+1)} = x^{(k)} - \gamma \nabla F(x^{(k)}; \xi^{(k)})$$

  - $\xi^{(k)}$ is the data sampled at iteration k
  - $\nabla F(x^{(k)}; \xi^{(k)})$ is a stochastic gradient associated with sample $\xi^{(k)}$
  - γ is the learning rate

Why does stochastic gradient descent work?

- If stochastic gradient is unbiased, i.e.,

$$\mathbb{E}_{\xi\sim D}\nabla F(x^{(k)};\xi) = \nabla\,\mathbb{E}_{\xi\sim D}[F(x^{(k)};\xi)] = \nabla f(x^{(k)}),$$

  the SGD recursion in expectation becomes

$$\begin{aligned}\mathbb{E}[x^{(k+1)}] &= \mathbb{E}[x^{(k)}] - \gamma\mathbb{E}[\nabla F(x^{(k)};\xi)]\\ &= \mathbb{E}[x^{(k)}] - \gamma\nabla f(x^{(k)}),\end{aligned}$$

  which reduces to a deterministic gradient descent.

- In other words, SGD is equivalent to GD in expectation. This intuitively explains why SGD works.

Stochastic gradient descent: convergence

Assumption

(A1) The loss function $F(x;\xi)$ is L-smooth in terms of x;

(A2) The stochastic gradient is unbiased, and has bounded variance σ^2.

Theorem

Under the above assumptions, and let $\gamma = O(1/\sqrt{T})$, we have

$$\frac{1}{T}\sum_{k=0}^{T-1}\mathbb{E}\|\nabla f(x^{(k)})\|^2 = O\left(\frac{\sigma}{\sqrt{T}}\right)$$

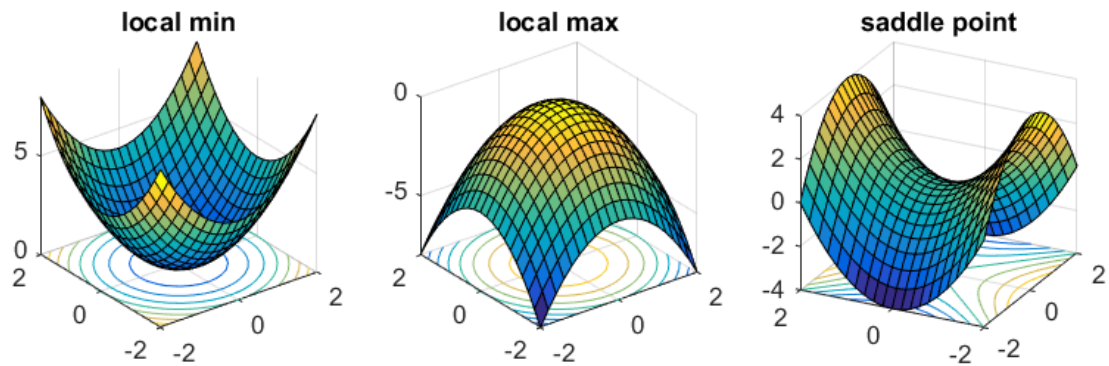
where $T \geq 1$ is the number of iterations

Note that we do no assume convexity for $f(x)$.

Stochastic gradient descent: convergence

$$\frac{1}{T}\sum_{k=0}^{T-1}\mathbb{E}\|\nabla f(x^{(k)})\|^2 = O\left(\frac{\sigma}{\sqrt{T}}\right)$$

- When iteration $T \to \infty$, it holds that $\mathbb{E}\|\nabla f(x^{(k)})\|^2 \to 0$
- $\mathbb{E}\|\nabla f(x^{(k)})\|^2 \to 0$ implies SGD converges to a stationary solution
- A stationary solution can be local min, local max, or saddle point[4]

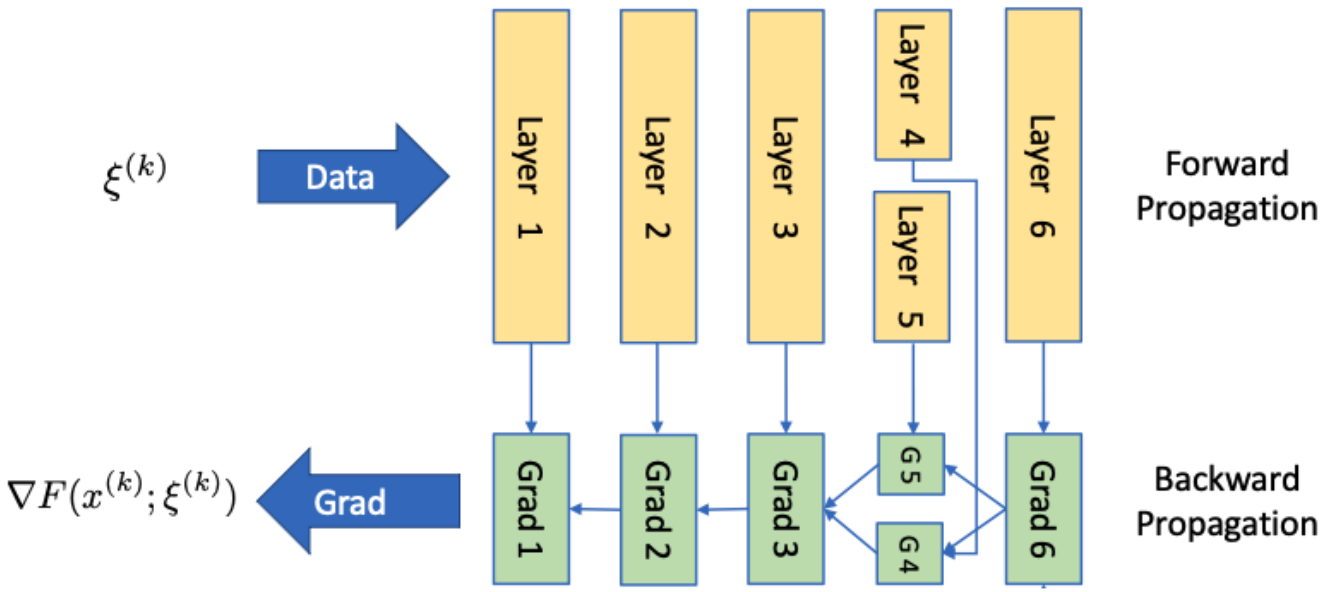


[4] Image source: from Prof. Rong Ge's online post

Stochastic gradient descent: convergence

- Generally speaking, approaching the stationary solution is the best result we can get for SGD; no guarantee to approach the global minimum
- Empirically, SGD performs extremely well when training DNN
- Recent advanced studies show SGD can escape local maximum, saddle point, and even "sharp" local minimum, see, e.g., (Ge et al., 2015; Sun et al., 2015; Jin et al., 2017; Du et al., 2018, 2019; Kleinberg et al., 2018) and references therein
- SGD can even find global minimum under certain conditions, e.g. the PL condition (Karimi et al., 2016)
- However, we will skip these exciting results in this lecture

Implementing SGD in DNN training



- Stochastic gradient can be calculated via forward-backward propagation
- Stochastic gradient can be achieved automatically via Pytorch/Tensorflow
- DNN training typically utilizes GPUs
- Momentum-SGD/ADAM are very useful to accelerate DNN training

Image Classification

- Cifar-10 dataset
- 50K training images
- 10K test images
- DNN model: ResNet-18
- GPU: Tesla V100

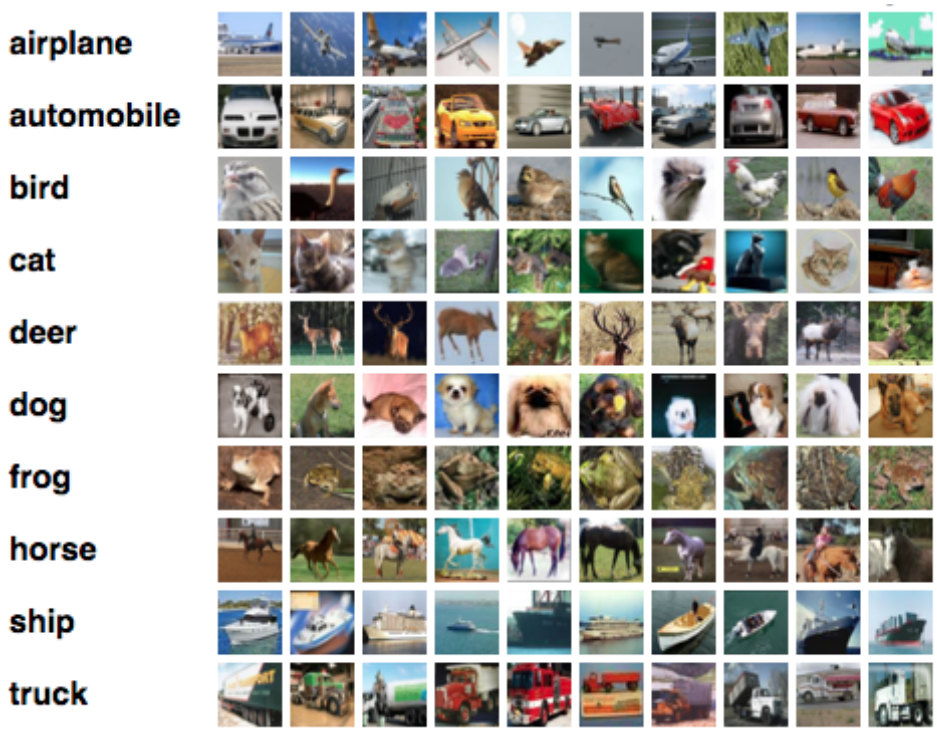
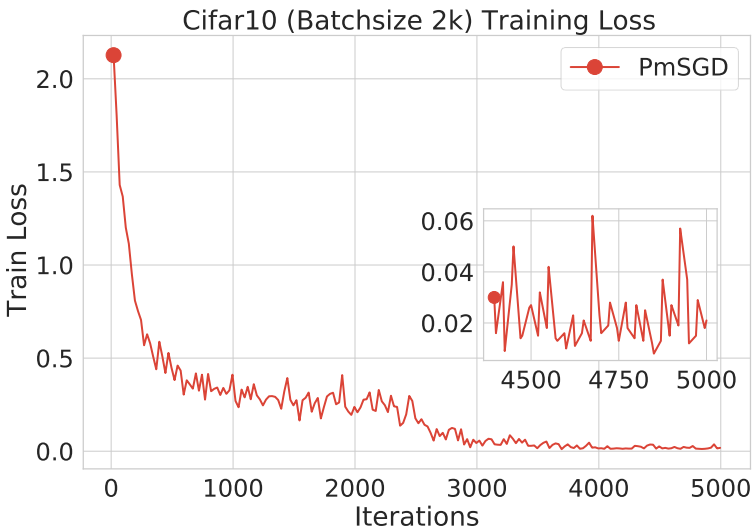
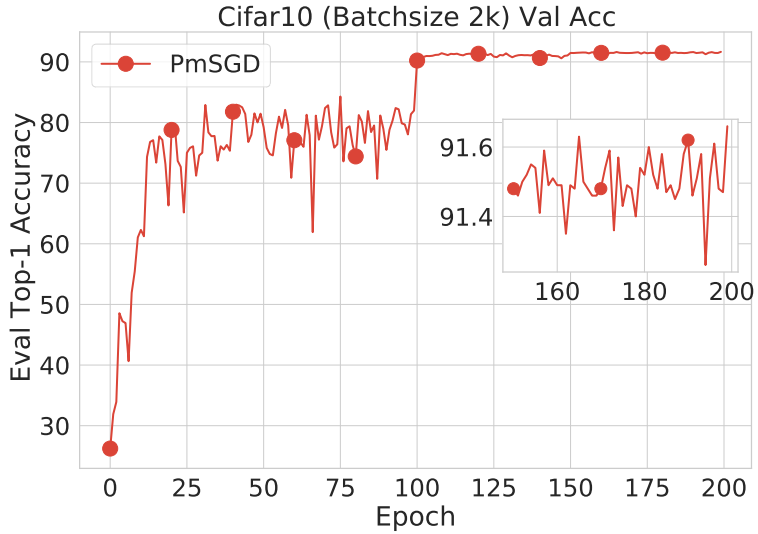


Image Classification





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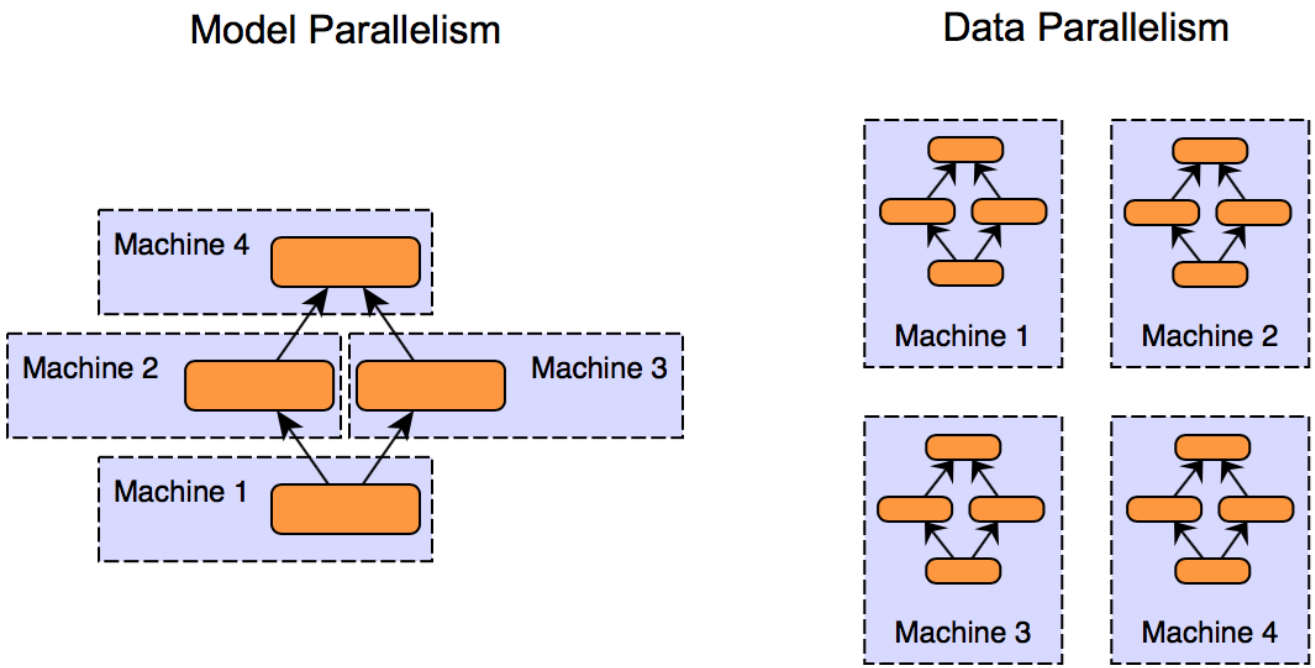
Parallel/Distributed training is necessary in DNN

- Scale to larger models and bigger data
- Bring down training time from days to hours
- Different types of parallel training:
  - Data-parallel training: share the model; partition the data
  - Model-parallel training: share the data; partition the model
  - Data-parallel and model-parallel mixed training
- In this lecture, we will focus on data-parallel training

Data-parallel and model-parallel training[5]



[5] Image source: https://xiandong79.github.io/Intro-Distributed-Deep-Learning

DNN training formulated as distributed optimization

- A network of n nodes (GPUs) collaborate to solve the problem:

$$\min_{x \in \mathbb{R}^d} \quad f(x) = \frac{1}{n} \sum_{i=1}^{n} [f_i(x) = \mathbb{E}_{\xi_i \sim D_i} F(x; \xi_i)].$$

- Each component $f_i : \mathbb{R}^d \to \mathbb{R}$ is local and private to node i
- Random variable ξ_i denotes the local data that follows distribution D_i
- Each local distribution D_i may be different; data heterogeneity

DNN training formulated as distributed optimization

- We consider deep training within high-performance **data-center** clusters
  - all GPUs are connected with high-bandwidth channels
  - network topology can be fully controlled
  - communication is highly reliable; no occasional link failure
- Different from the mobile AI applications, or Federated Learning where
  - nodes are connected with low-bandwidth channels
  - network topology can not be controlled
  - communication is highly fragile; occasional link failures

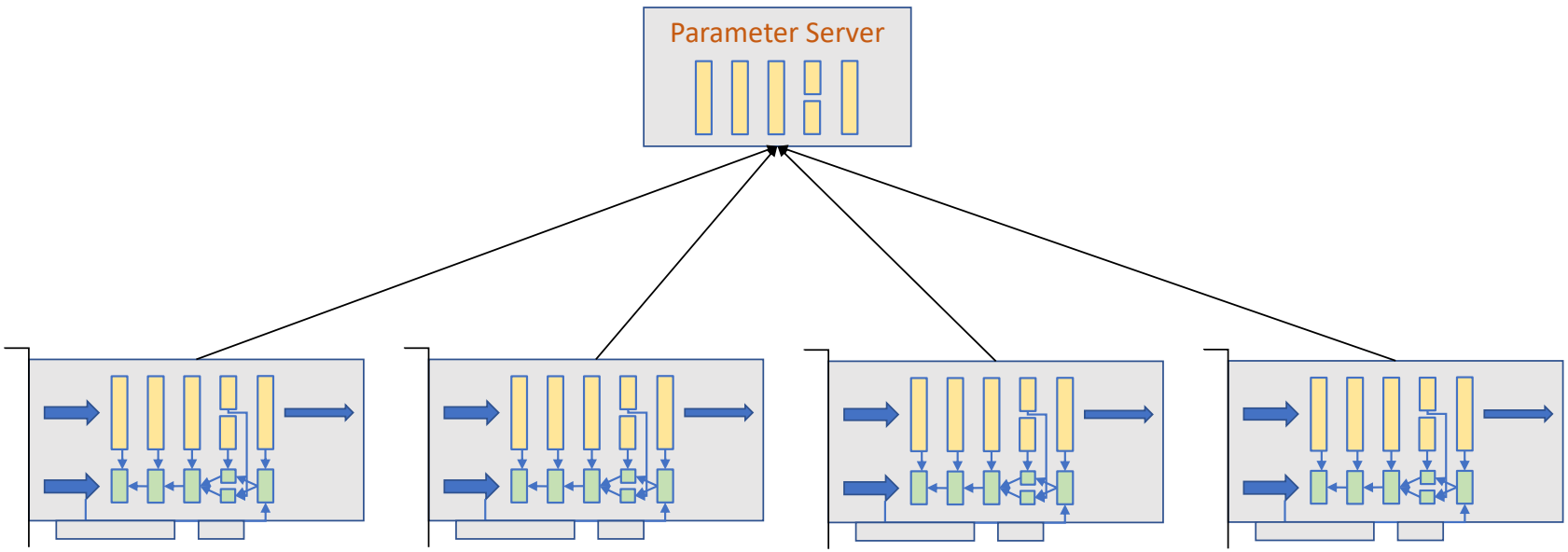
Parallel stochastic gradient descent (SGD)

$$g_i^{(k)} = \nabla F(x^{(k)}; \xi_i^{(k)}) \qquad \text{(Local compt.)}$$

$$x^{(k+1)} = x^{(k)} - \frac{\gamma}{n} \sum_{i=1}^{n} g_i^{(k)} \qquad \text{(Global comm.)}$$

- Each node i samples data $\xi_i^{(k)}$ and computes gradient $\nabla F(x^{(k)}; \xi_i^{(k)})$
- All nodes synchronize (i.e. global averaged) to update model x
- Global average incurs significant comm. cost; hinders training scalability

Global average via Parameter-Server (Li et al., 2014)



Global average via Ring-Allreduce (Patarasuk and Yuan, 2009)

Parallel SGD convergence

Assumption

(A1) Each local loss function $F(x;\xi_i)$ is L-smooth in terms of x;

(A2) Each local stochastic gradient is unbiased, and has bounded variance σ^2:

$$\mathbb{E}[g_i^{(k)}] = \nabla f_i(x^{(k)}), \quad \mathbb{E}\|g_i^{(k)} - \nabla f_i(x^{(k)})\|^2 \leq \sigma^2$$

(A3) Each local stochastic gradient $g_i^{(k)}$ is independent of each other

The variance of the globally averaged gradient is remarkably reduced:

$$\mathbb{E}\|\frac{1}{n}\sum_{i=1}^{n} g_i^{(k)} - \nabla f(x^{(k)})\|^2 = \frac{1}{n^2}\sum_{i=1}^{n} \mathbb{E}\|g_i^{(k)} - \nabla f_i(x^{(k)})\|^2 \leq \frac{\sigma^2}{n}$$

Parallel SGD (P-SGD) convergence

- Substituting the above inequality into the derivation, we achieve

Theorem (Parallel SGD convergence)

Under the above assumptions, and let $\gamma = O(1/\sqrt{T})$*, we have*

$$\frac{1}{T}\sum_{k=0}^{T-1}\mathbb{E}\|\nabla f(x^{(k)})\|^2 = O\left(\frac{\sigma}{\sqrt{nT}}\right)$$

where $T \geq 1$ *is the number of iterations,* n *is the number of nodes.*

- We achieve single-node SGD convergence when $n = 1$

Parallel SGD can achieve linear speedup

- Recall the SGD convergence rate:

$$\text{Single-node training:} \quad \frac{1}{T}\sum_{k=0}^{T-1}\mathbb{E}\|\nabla f(x^{(k)})\|^2 = O\left(\frac{\sigma}{\sqrt{T}}\right)$$

$$n\text{-node parallel training:} \quad \frac{1}{T}\sum_{k=0}^{T-1}\mathbb{E}\|\nabla f(x^{(k)})\|^2 = O\left(\frac{\sigma}{\sqrt{nT}}\right)$$

- To achieve an ϵ-accurate solution, i.e., $\frac{1}{T}\sum_{k=0}^{T-1}\mathbb{E}\|\nabla f(x^{(k)})\|^2 \leq \epsilon$,

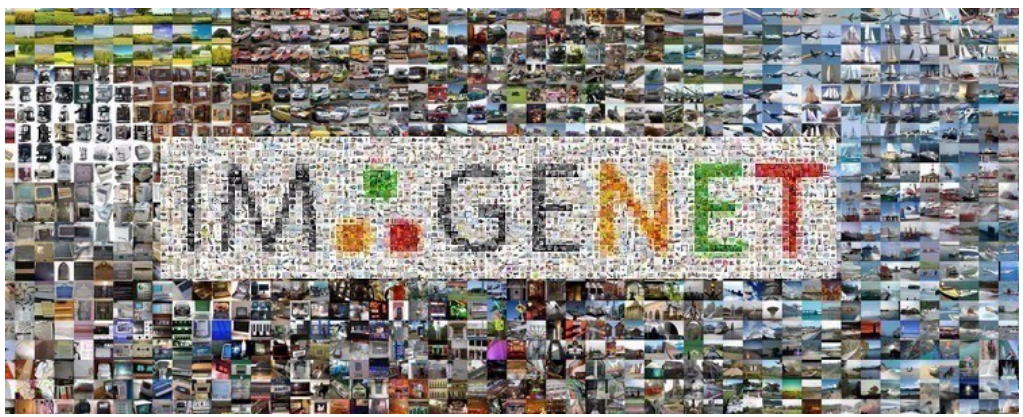
$$\text{Single-node training requires} \quad \frac{\sigma^2}{\epsilon^2} \quad \text{iterations}$$

$$n\text{-node parallel training requires} \quad \frac{\sigma^2}{n\epsilon^2} \quad \text{iterations}$$

- Iteration complexity is inversely proportional to n; P-SGD has linear speedup

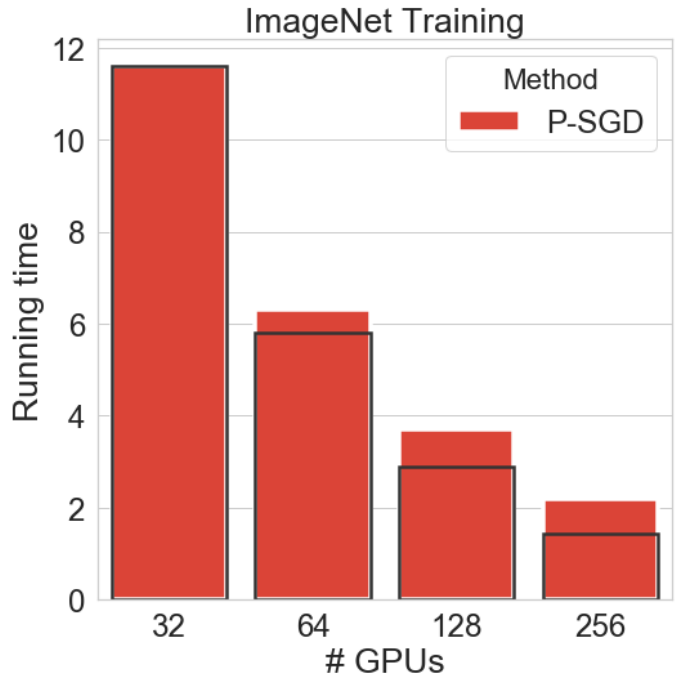
Image Classification

- ImageNet-1K dataset
- 1.3M training images
- 50K test images
- 1K classes
- DNN Model: ResNet-50 (∼25.5M parameters)
- GPU: Tesla V100 clusters
- Framework: Pytorch DDP



Parallel SGD has linear speedup in DNN training

- Wall-clock training time to achieve $> 76\%$ top-1 accuracy (black box indicates ideal running time linear speedup)



- Cannot achieve ideal linear speedup due to comm. cost
- Global average incurs significant comm. cost; hinders training scalability

Comm. overhead in global average

- A single communication includes **bandwidth cost** and **latency** (Ben-Nun and Hoefler, 2019)
- The single communication cost

	Bandwidth Cost	Latency	Total Cost
Parameter server	$\Omega(n)$	$\Omega(1)$	$\Omega(n+1)$
Ring allreduce	$\Omega(1)$	$\Omega(n)$	$\Omega(1+n)$

- In either approach, the cost is $\Omega(n)$, proportional to network size n.
- In deep training, the bandwidth cost is typically more severe; but latency cannot be ignored neither
- To approach the ideal linear speedup, comm. cost must be reduced

Approaches to saving communication cost

- Model/Gradient sparsification (Tang et al., 2019; Koloskova et al., 2019a,b; Wangni et al., 2017; Alistarh et al., 2018; Stich et al., 2018)
- Model/Gradient quantization (Das et al., 2018; Alistarh et al., 2017; Bernstein et al., 2018; Wen et al., 2017)
- Local SGD/lazy-communication (Chen et al., 2018; Liu et al., 2019; Chen et al., 2020; Zinkevich et al., 2010; Zhang et al., 2016; Stich, 2019; Yu et al., 2019a,b; Lin et al., 2018; McMahan et al., 2017; Li et al., 2019a)
- Decentralized communication (Lopes and Sayed, 2008; Nedic and Ozdaglar, 2009; Shi et al., 2015; Yuan et al., 2016; Assran et al., 2019; Yuan et al., 2019; Li et al., 2019b; Di Lorenzo and Scutari, 2016; Nedic et al., 2017; Qu and Li, 2018)

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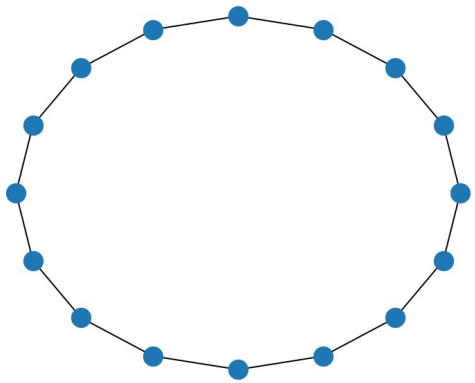
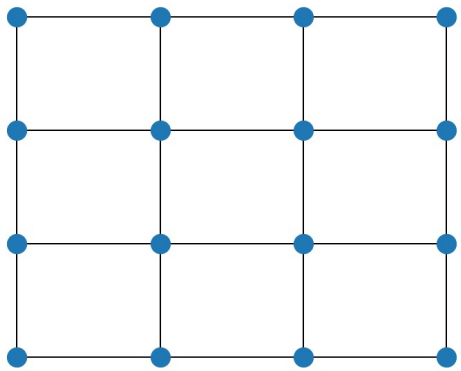
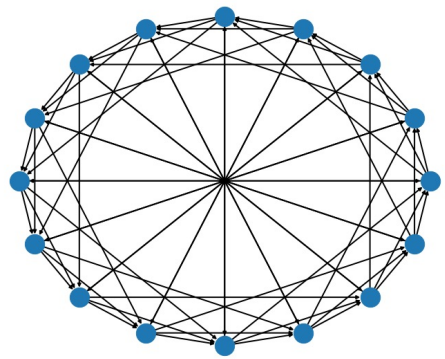
Decentralized SGD: topology

- Assume we connect all nodes with some topology (n=16)







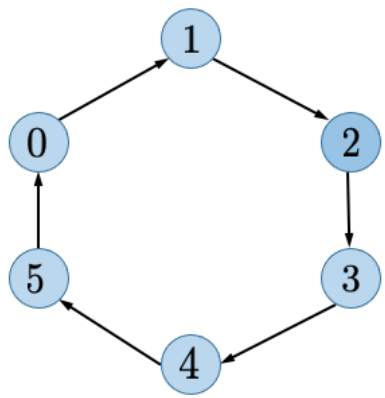
- Communication is only allowed between neighbors
- No global synchronization is allowed

Decentralized SGD: weight matrix

- The weight matrix associated with the topology is defined as

$$w_{ij} \begin{cases} > 0 & \text{if node } j \text{ is connected to } i, \text{ or } i = j; \\ = 0 & \text{otherwise.} \end{cases}$$

- Throughout the lecture we assume the row and column sums of W to be 1
- An example:



$$W = \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Figure: A directed ring topology and its associated combination matrix W.

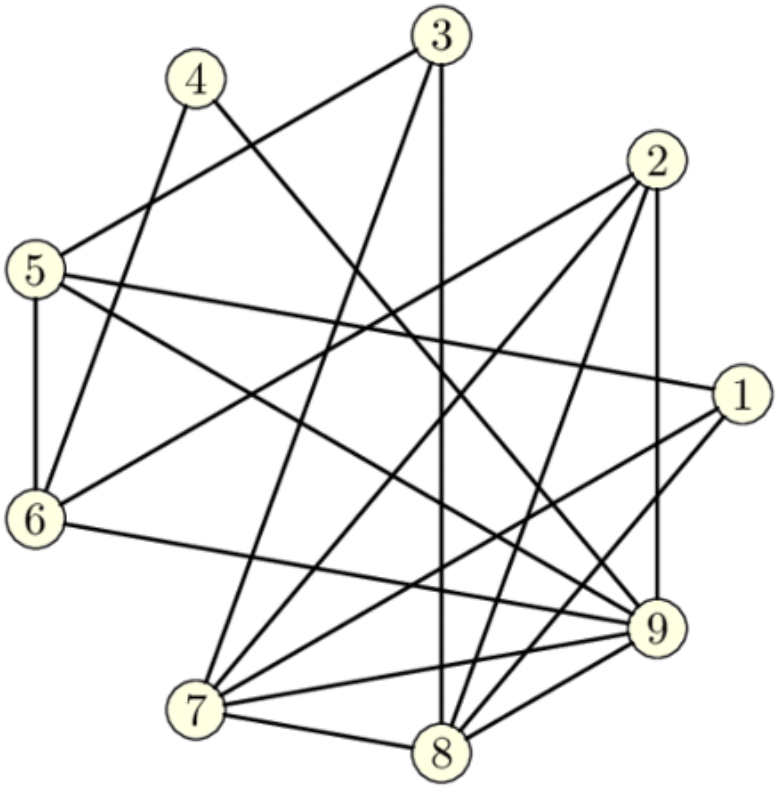
Decentralized SGD (D-SGD): partial averaging

- D-SGD is based on partial-averaging within neighborhood

$$\text{Partial averaging:} \quad x_i^+ \leftarrow \sum_{j \in \mathcal{N}_i} w_{ij} x_j. \quad \forall i \in [n]$$

- $\mathcal{N}_i$ is the set of neighbors of node i
- Each node only communicates with neighbors; no global sync
- Incurs $\Omega(d_{\max})$ comm. overhead ($d_{\max}$: maximum degree)

Maximum degree[6]



$$d_1 = 3$$
$$d_2 = 4$$
$$d_3 = 3$$
$$\vdots$$
$$d_9 = 6$$

$$d_{\max} = \max_i \{d_i\} = 6$$

[6] Image source:

Decentralized SGD (D-SGD): recursions

- D-SGD = local SGD update+ paritial averaging (Loizou and Richtárik, 2020; Nedic and Ozdaglar, 2009; Chen and Sayed, 2012)

$$x_i^{(k+\frac{1}{2})} = x_i^{(k)} - \gamma \nabla F(x_i^{(k)}; \xi_i^{(k)}) \quad \text{(Local update)}$$

$$x_i^{(k+1)} = \sum_{j \in \mathcal{N}_i} w_{ij} x_j^{(k+\frac{1}{2})} \quad \text{(Partial averaging)}$$

- Per-iteration communication: $\Omega(d_{\max}) \ll \Omega(n)$ when topology is sparse
- Incurs $\Omega(1)$ comm. overhead on sparse topology (ring or grid)

Decentralized SGD is more communication efficient

Model	Ring-Allreduce	Partial average
ResNet-50	278 ms	150 ms
Bert	1469 ms	567 ms

Table: Comparison of per-iter comm. in terms of runtime with 256 GPUs

- ResNet-50 has 25.5M parameters; Bert has 300M parameters
- Partial average saves more communication for larger model

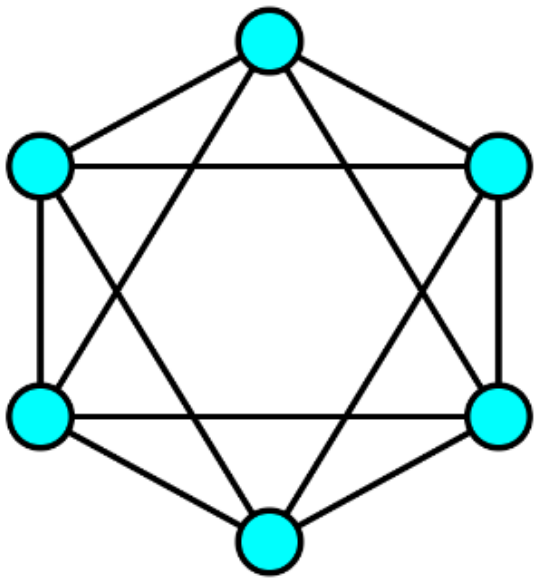
However, D-SGD has slower convergence

- The efficient communication comes with a cost: slow convergence
- Partial averaging is less effective to aggregate information
- The average effectiveness can be evaluated by spectral gap:

$$\rho = \|W - \frac{1}{n}\mathbb{1}\mathbb{1}^T\|_2$$

  - Assume W is doubly-stochastic, it holds that $\rho \in (0, 1)$.
  - Well-connected topology has $\rho \to 0$, e.g. fully-connected topology
  - Sparsely-connected topology has $\rho \to 1$, e.g., ring has $\rho = O(1 - \frac{1}{n^2})$

Weight-matrix of the fully-connected topology



$$W = \frac{1}{5}\mathbb{1}\mathbb{1}^T = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \end{bmatrix}$$

Decentralized SGD convergence

Recall the assumptions of P-SGD:

Assumption

(A1) Each local loss function $F(x;\xi_i)$ is L-smooth in terms of x;

(A2) Each local stochastic gradient is unbiased, and has bounded variance σ^2:

$$\mathbb{E}[g_i^{(k)}] = \nabla f_i(x^{(k)}), \quad \mathbb{E}\|g_i^{(k)} - \nabla f_i(x^{(k)})\|^2 \leq \sigma^2$$

(A3) Each local stochastic gradient $g_i^{(k)}$ is independent of each other

We further introduce another data-heterogeneity assumption

Assumption

(A4) The data heterogeneity is bounded, i.e.,

$$\frac{1}{n}\sum_{i=1}^{n}\|\nabla f_i(x) - \nabla f(x)\|^2 \leq b^2, \quad \forall x \in \mathbb{R}^d$$

When D_i is identical, we have $\nabla f_i(x) = \nabla f(x)$ for any i and hence $b^2 = 0$

Decentralized SGD convergence

- (Lian et al., 2017; Assran et al., 2019; Koloskova et al., 2020) show that

Theorem (Decentralized SGD convergence)

Under Assumptions (A1)-(A4), and let $\gamma = O(1/\sqrt{T})$*, we have*

$$\frac{1}{T}\sum_{k=0}^{T-1}\mathbb{E}\|\nabla f(x^{(k)})\|^2 = O\left(\frac{\sigma}{\sqrt{nT}} + \frac{\rho^{2/3}\sigma^{2/3}}{T^{2/3}(1-\rho)^{1/3}} + \frac{\rho^{2/3}b^{2/3}}{T^{2/3}(1-\rho)^{2/3}}\right)$$

where $T \geq 1$ *is the number of iterations, and* n *is the number of nodes.*

- When topology is fully connected ($\rho = 0$), D-SGD reduces to P-SGD.
- When $\rho = 0$ and $n = 1$, D-SGD reduces to single-node SGD

Convergence rate: P-SGD v.s. D-SGD

- Convergence comparison (i.i.d data distribution, i.e., $b^2 = 0$):

$$\text{P-SGD}: \quad \frac{1}{T}\sum_{k=1}^{T}\mathbb{E}\|\nabla f(\bar{x}^{(k)})\|^2 = O\Big(\frac{\sigma}{\sqrt{nT}}\Big)$$

$$\text{D-SGD}: \quad \frac{1}{T}\sum_{k=1}^{T}\mathbb{E}\|\nabla f(\bar{x}^{(k)})\|^2 = O\Big(\frac{\sigma}{\sqrt{nT}} + \underbrace{\frac{\rho^{2/3}\sigma^{2/3}}{T^{2/3}(1-\rho)^{1/3}}}_{\text{extra overhead}}\Big)$$

  where σ^2 is the gradient noise, and T is the number of iterations.

- D-SGD can asymptotically converge as fast as P-SGD when $T \to \infty$; the first term dominates; reach linear speedup asymptotically

- But it requires more iteration (i.e., T has to be large enough) to reach that stage due to the extra overhead caused by partial averaging

Transient iterations

- **Definition** (Pu et al., 2020): number of iterations before D-SGD achieves linear speedup
- Transient iterations measure the converg. gap between P-SGD and D-SGD
- Longer tran. iters. $\Longrightarrow$ slower convergence than P-SGD
- The transient iteration complexity of D-SGD is

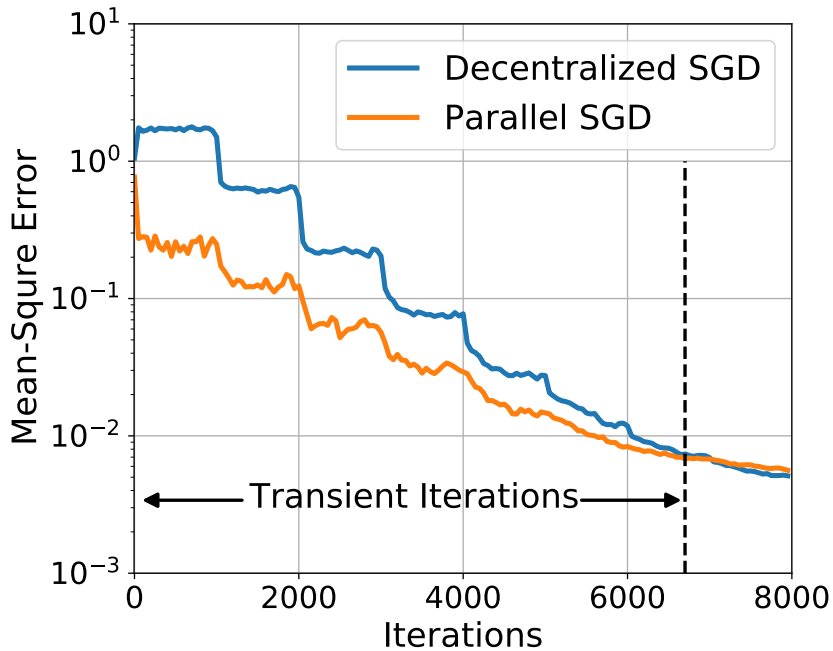
$$\text{iid data}: \quad \frac{\rho^{2/3}\sigma^{2/3}}{T^{2/3}(1-\rho)^{1/3}} \leq \frac{\sigma}{\sqrt{nT}} \quad \Longrightarrow \quad T = \Omega\left(\frac{\rho^4 n^3}{(1-\rho)^2}\right)$$

$$\text{non-iid data}: \quad \frac{\rho^{2/3}b^{2/3}}{T^{2/3}(1-\rho)^{2/3}} \leq \frac{\sigma}{\sqrt{nT}} \quad \Longrightarrow \quad T = \Omega\left(\frac{\rho^4 n^3}{(1-\rho)^4}\right)$$

- Sparse topology ($\rho \to 1$) incurs large tran. iters. complexity

Transient iterations: illustration

Illustration of the tran. iters. on D-SGD over ring (logistic regression)



If the transient stage is too long, we may not be able to achieve linear speedup given the limited time/resource budget

Part I summary

- DNN training can be formulated as stochastic optimization
- SGD is the leading approach to train DNN
- Parallel SGD can achieve linear speedup theoretically; but the comm. cost incurred by global average hinders its empirical linear speedup performance
- Decentralized SGD utilizes partial averaging within neighborhood; reduce per-iter comm. cost from $\Omega(n)$ to $\Omega(d_{\max})$, and even $\Omega(1)$.
- D-SGD suffers from slower convergence; compensate its comm. efficiency.

In Part II, we will

Introduce several techniques to accelerate D-SGD and make it practically valuable for large-scale deep learning

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