Decentralized Optimization Algorithms for Large-Scale Deep Neural Network Training

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- Single-node training
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- Primal-dual decentralized methods
- Multiple gossip loops/Periodic global averaging

Other advanced topics and BlueFog (Part III)

- Large-batch deep training
- An open source decentralized deep training framework: BlueFog

Making decentralized methods practical: review

Which topology shall we use to organize all GPUs?

Topology	Per-iter. Comm.	Trans. Iters. (iid scenario)
Ring	$\Omega(2)$	$\Omega(n^7)$
Star	$\Omega(n)$	$\Omega(n^7)$
2D-Grid	$\Omega(4)$	$\Omega(n^5 \log_2^2(n))$
2D-Torus	$\Omega(4)$	$\Omega(n^5)$
$\frac{1}{2}$-RandGraph	$\Omega(\frac{n}{2})$	$\Omega(n^3)$
Static Exp.	$\tilde{\Omega}(1)$	$\tilde{\Omega}(n^3)$
One-peer Exp.	$\Omega(1)$	$\tilde{\Omega}(n^3)$

Making decentralized methods practical: review

How to accelerate D-SGD when non-iid data exists? Exact-Diffusion

non-iid scenario	Exact-Diffusion	D-SGD
strongly-convex	$\Omega(\frac{\rho^2 n}{1-\rho})$	$\Omega(\frac{\rho^2 n}{(1-\rho)^2})$
generally-convex	$\Omega(\frac{\rho^4 n^3}{(1-\rho)^2})$	$\Omega(\frac{\rho^4 n^3}{(1-\rho)^4})$
non-convex	N.A.	$\Omega(\frac{\rho^4 n^3}{(1-\rho)^4})$

Making decentralized methods practical: review

How to accelerate D-SGD over extremely sparse topology (i.e., $\rho \to 1$)?

Decentralized SGD with Periodic Global Averaging

scenario	DSGD-PGA	D-SGD
iid data	$\Omega(\rho^4 n^3 H^2)$	$\Omega(\frac{\rho^4 n^3}{(1-\rho)^2})$
non-iid data	$\Omega(\rho^4 n^3 H^4)$	$\Omega(\frac{\rho^4 n^3}{(1-\rho)^4})$

Part III: Other advanced topics and BlueFog

- Sec. 1 Large-batch deep training
- Sec. 2 An open source decentralized deep training framework: BlueFog

Advanced topics

- Decentralized DL with directed topology (Assran et al., 2019; Pu et al., 2020; Xin and Khan, 2018)
- Decentralized DL with time-varying topology (Koloskova et al., 2020; Nedic et al., 2017)
- Decentralized DL with severe data heterogeneity (Tang et al., 2018a; Lin et al., 2021; Xin et al., 2020; Lu et al., 2019)
- Decentralized DL with asynchrony and delays (Lian et al., 2018; Zhang and You, 2019; Wu et al., 2017)
- Decentralized DL with compression and quantization (Koloskova et al., 2019a,b; Tang et al., 2018b; Liu et al., 2020; Kovalev et al., 2021)

Unfortunately we cannot cover these topics in this lecture.

But let's discuss an important topic that is easy to be ignored:

Decentralized large-batch deep training

Motivation

- Total batch size increases as the number of nodes (GPUs) increase
- Suppose each GPU takes 256 samples per iteration:

$$\text{(8 GPUs:)} \quad 256 \times 8 = 2K \quad \text{(samples)}$$
$$\text{(64 Gpus:)} \quad 256 \times 64 = 16K \quad \text{(samples)}$$

- Large-batch training is unavoidable when more nodes participate in

Decentralized momentum SGD

- Recall the distributed optimization problem

$$\min_{x \in \mathbb{R}^d} \quad f(x) = \frac{1}{n} \sum_{i=1}^{n} [f_i(x) = \mathbb{E}_{\xi_i \sim D_i} F(x; \xi_i)].$$

- Recall the D-SGD algorithm

$$x_i^{(k+\frac{1}{2})} = x_i^{(k)} - \gamma \nabla F(x_i^{(k)}; \xi_i^{(k)}) \quad \text{(Local update)}$$

$$x_i^{(k+1)} = \sum_{j \in \mathcal{N}_i} w_{ij} x_j^{(k+\frac{1}{2})} \quad \text{(Partial averaging)}$$

- The **momentum accelerated** D-SGD is more popular in real deep learning

Decentralized momentum SGD

Algorithm 1: DmSGD

Require: Initialize γ, $x_i^{(0)}$; let $m_i^{(0)} = 0, \beta \in (0, 1)$

for $k = 0, 1, 2, ..., T-1$, *every node* i **do**

Sample $\xi_i^{(k)}$ and update $g_i^{(k)} = \nabla F(x_i^{(k)}; \xi_i^{(k)})$

$m_i^{(k+1)} = \beta m_i^{(k)} + g_i^{(k)}$ ▷ momentum update

$x_i^{(k+\frac{1}{2})} = x_i^{(k)} - \gamma m_i^{(k+1)}$ ▷ local model update

$x_i^{(k+1)} = \sum_{j \in \mathcal{N}_i} w_{ij} x_j^{(k+\frac{1}{2})}$ ▷ partial average

---

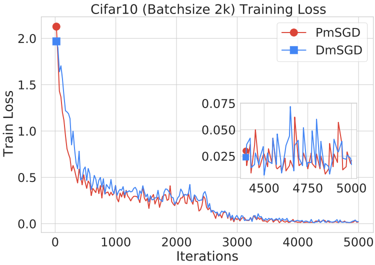
- The above DmSGD method is widely used in decentralized deep training[1]
- Reduces to D-SGD when $\beta = 0$

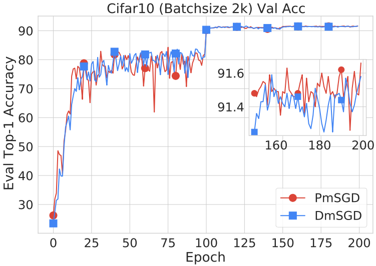
[1] [Lian et.al., 2018; Assran et.al., 2019; Gao and Huang, 2020]

Large-batch DmSGD has poor performance

Experimental setting: CIFAR-10; ResNet-20

Small-batch: 2K batch-size per iteration



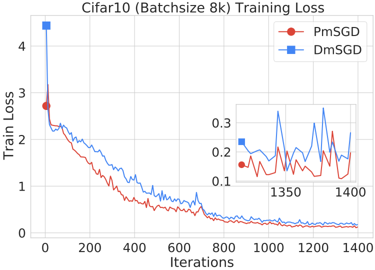


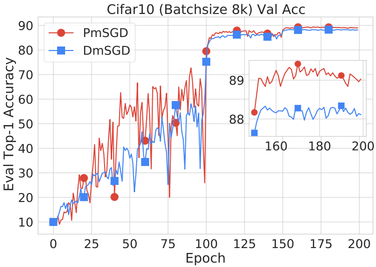
DmSGD and PmSGD have almost the **same** performance with small-batch.

Large-batch DmSGD has poor performance

Experimental setting: CIFAR-10; ResNet-20

Large-batch setting: 8K batch-size per iteration





DmSGD **drops** 1% performance compared to PmSGD with large-batch.

Why does DmSGD have severe performance degradation than PmSGD?

Limiting bias of DmSGD

The limiting bias of DSGD/DmSGD (s.c. cost) suffers from two sources:

$$\lim_{k\to\infty}\sum_{i=1}^{n}\mathbb{E}\|x_i^{(k)}-x^\star\|^2 = \text{sto. bias} + \text{inconsist. bias}$$

- stochastic bias is cuased by the gradient noise
- inconsistency bias is caused by the data heterogeneity (different D_i)
- As batch-size increases, sto. bias will vanish and incost. bias will dominate

Proposition

The inconsistency bias dominates the convergence of large-batch *DmSGD.*

Limiting bias of DmSGD

For example, the limiting bias of DSGD (s.c. cost) is (Yuan et al., 2020):

$$\lim_{k\to\infty}\sum_{i=1}^{n}\mathbb{E}\|x_i^{(k)}-x^\star\|^2 = O\Big(\underbrace{\frac{\gamma^2\sigma^2}{n}+\frac{\gamma^2\sigma^2}{1-\rho}}_{\text{sto. bias}}+\underbrace{\frac{\gamma^2 b^2}{(1-\rho)^2}}_{\text{inconsist. bias}}\Big)$$

- where $b^2 = \frac{1}{n}\sum_{i=1}^{n}\|\nabla f_i(x^\star)\|^2$ denotes data heterogeneity
- when each D_i is identical, it holds that $f_i(x) = f_j(x)$ for any i and j, which implies that $\nabla f_i(x^\star) = 0$ and hence $b^2 = 0$
- in other words, $b^2 = 0$ for when each D_i is identical (i.i.d. scenario)
- $\sigma^2 \to 0$ as batch-size goes large, and hence inconsist. bias dominates

DmSGD incurs severe inconsistency bias

We rewrite full-batch DmSGD recursion as follows:

$$x_i^{(k+1)} = \underbrace{\sum_{j \in \mathcal{N}_i} w_{ij}\Big(x_j^{(k)} - \gamma \nabla f_j(x_j^{(k)})\Big)}_{\text{DSGD}} \quad \text{(DmSGD)}$$
$$+ \underbrace{\beta\Big(x_i^{(k)} - \sum_{j \in \mathcal{N}_i} w_{ij} x_j^{(k-1)}\Big)}_{\text{momentum}}, \ \forall i \in [n].$$

- No stochastic bias in the above recursion (full-batch gradient)
- momentum will not vanish as $x_i^{(k)} \neq \sum_{j \in \mathcal{N}_i} w_{ij} x_j^{(k-1)}$ as $k \to \infty$;
- momentum will incur additional inconsistency bias.

DmSGD incurs severe inconsistency bias

Proposition (Yuan et al. (2021))

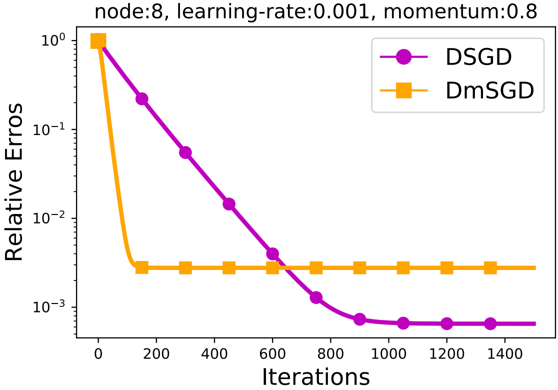
The full-batch DmSGD (S.C. cost) has the following inconsistency bias:

$$\lim_{k\to\infty}\sum_{i=1}^{n}\|x_i^{(k)}-x^\star\|^2 = O\Big(\frac{\gamma^2 b^2}{(1-\beta)^2(1-\rho)^2}\Big),$$

where $b^2=(1/n)\sum_{i=1}^{n}\|\nabla f_i(x^\star)\|^2$ *denotes the data inconsistency between nodes, and* β *is the momentum coefficient.*

- Recall that full-batch D-SGD has limiting bias $O(\gamma^2 b^2/(1-\rho)^2)$
- The momentum in DmSGD amplifies the inconsistency bias as $\beta\in(0,1)$
- DmSGD suffers from significant inconsist. bias when $\beta\to 1$
- Such amplified inconsist. bias results in notable performance degradation in large-batch scenario

DmSGD incurs severe inconsistency bias: verification



- A numerical verification: full-batch linear regression
- DmSGD is faster but suffers from more inconsistency bias (as expected)

Remove the momentum-incurred bias

We modify the full-batch DmSGD a little bit (Yuan et al., 2021)[2]:

$$x_i^{(k+1)} = \underbrace{\sum_{j \in \mathcal{N}_i} w_{ij} \Big(x_j^{(k)} - \gamma \nabla f_j(x_j^{(k)}) \Big)}_{\text{DSGD}} \quad \text{(DecentLaM)}$$

$$+ \underbrace{\beta \Big(x_i^{(k)} - x_i^{(k-1)} \Big)}_{\text{momentum}}, \ \forall i \in [n].$$

- $x_i^{(k)} - x_i^{(k-1)} \to 0$ as $k \to \infty$;
- momentum-incurred bias will vanish as $k \to \infty$;
- we name the above algorithm as full-bath DecentLaM

[2] K. Yuan, Y. Chen, X. Huang, Y. Zhang, P. Pan, Y. Xu, and W. Yin, "DecentLaM: Decentralized Stochastic Momentum SGD for Large-batch Deep Training", to appear in ICCV 2021

Another useful algorithm derivation

- We let $\mathbf{x} = [x_1, \cdots, x_n]^T \in \mathbb{R}^{n\times d}$ and $f(\mathbf{x}) = \sum_{i=1}^{n} f_i(x_i)$
- We assume W is positive-definite and doubly stochastic, and $f_i(x)$ is s.c.
- We introduce $\mathbf{s} = W^{-\frac{1}{2}}\mathbf{x}$ and hence $\mathbf{x} = W^{\frac{1}{2}}\mathbf{s}$
- The full-batch DSGD algorithm can be rewritten as

$$
\begin{aligned}
& \mathbf{x}^{(k+1)} = W(\mathbf{x}^{(k)} - \gamma \nabla f(\mathbf{x}^{(k)})) \\
\Longleftrightarrow \quad & \mathbf{s}^{(k+1)} = W\mathbf{s}^{(k)} - \gamma W^{\frac{1}{2}} \nabla f(W^{\frac{1}{2}}\mathbf{s}^{(k)}) \\
& \qquad\quad = W\mathbf{s}^{(k)} - \gamma \nabla_{\mathbf{s}} f(W^{\frac{1}{2}}\mathbf{s}^{(k)}) \\
& \qquad\quad = \mathbf{s}^{(k)} - \gamma \underbrace{\left(\nabla_{\mathbf{s}} f(W^{\frac{1}{2}}\mathbf{s}^{(k)}) - \frac{1}{\gamma}(I - W)\mathbf{s}^{(k)}\right)}_{\text{gradient}}
\end{aligned}
$$

Another useful algorithm derivation

- We conclude that DSGD is essentially a standard GD for problem

$$\min_{\mathbf{s}} \quad f(W^{\frac{1}{2}}\mathbf{s}^{(k)}) + \frac{1}{2\gamma}\|\mathbf{s}\|_{I-W}^2$$

- When $\mathbf{s}^\star$ is achieved, we can derive $\mathbf{x}^\star = W^{\frac{1}{2}}\mathbf{s}^\star$

- Interpret DSGD as GD is critical; many techniques used in GD (such as momentum acceleration) can also be integrated to DSGD

- Add momentum to DSGD is equivalent to add momentum to GD:

$$\begin{aligned}
\mathbf{g}_{\mathbf{s}}^{(k)} &= \nabla_{\mathbf{s}} f(W^{\frac{1}{2}}\mathbf{s}^{(k)}) - \frac{1}{\gamma}(I-W)\mathbf{s}^{(k)} \\
\mathbf{m}_{\mathbf{s}}^{(k+1)} &= \beta \mathbf{m}_{\mathbf{s}}^{(k)} + \mathbf{g}_{\mathbf{s}}^{(k)} \\
\mathbf{s}^{(k+1)} &= \mathbf{s}^{(k)} - \gamma \mathbf{m}_{\mathbf{s}}^{(k+1)} \\
\mathbf{x}^{(k+1)} &= W^{\frac{1}{2}}\mathbf{s}^{(k+1)}
\end{aligned}$$

Another useful algorithm derivation

- Simplify the above recursions, we achieve

$$\begin{aligned}
\mathbf{g}^{(k)} &= \nabla f(\mathbf{x}^{(k)}) - \frac{1}{\gamma}(I - W)\mathbf{x}^{(k)} \\
\mathbf{m}^{(k+1)} &= \beta \mathbf{m}^{(k)} + \mathbf{g}^{(k)} \\
\mathbf{x}^{(k+1)} &= \mathbf{x}^{(k)} - \gamma \mathbf{m}^{(k+1)}
\end{aligned}$$

- Combining all recursions, we achieve

$$\mathbf{x}^{(k+1)} = W(\mathbf{x}^{(k)} - \nabla f(\mathbf{x}^{(k)})) + \beta(\mathbf{x}^{(k)} - \mathbf{x}^{(k-1)})$$

DecentLaM algorithm with stochastic gradient

Algorithm 2: DecentLaM

Require: Initialize γ, $x_i^{(0)}$; let $m_i^{(0)} = 0, \beta \in (0,1)$

for $k = 0, 1, 2, ..., T-1$, *every node* i **do**

  Sample $\xi_i^{(k)}$ and update $g_i^{(k)}$ according to (10)

  $m_i^{(k+1)} = \beta m_i^{(k)} + g_i^{(k)}$ ▷ momentum update

  $x_i^{(k+1)} = x_i^{(k)} - \gamma m_i^{(k+1)}$ ▷ local model update

where $g_i^{(k)}$ is computed as follows:

$$g_i^{(k)} = \frac{1}{\gamma} x_i^{(k)} - \frac{1}{\gamma} \sum_{j \in \mathcal{N}_i} w_{ij} \left(x_j^{(k)} - \gamma \nabla F(x_j^{(k)}; \xi_j^{(k)}) \right)$$

Remove the momentum-incurred bias

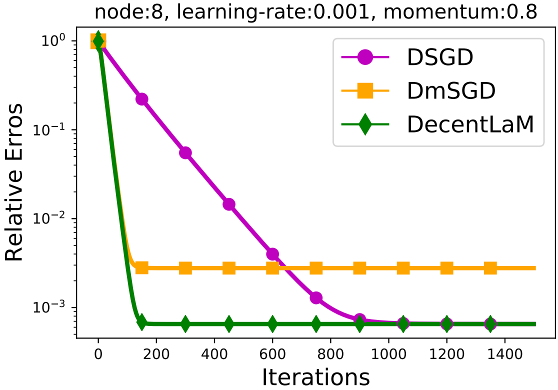
Proposition (Yuan et al. (2021))

Full-batch DecentLaM (S.C. cost) has an inconsistency bias as follows:

$$\lim_{k\to\infty}\sum_{i=1}^{n}\|x_i^{(k)}-x^\star\|^2 = O\Big(\frac{\gamma^2 b^2}{(1-\rho)^2}\Big),$$

- Recall that full-batch DmSGD has limiting bias $O(\frac{\gamma^2 b^2}{(1-\rho)^2(1-\beta)^2})$
- DecentLaM corrects the momentum-incurred bias
- DecentLaM has evident superiority when b^2 is large, or $\beta\to 1$, or $\rho\to 1$
- With smaller inconsist. bias, DecentLaM is expected to outperform DmSGD in large-batch scenario

Remove the momentum-incurred bias: verification

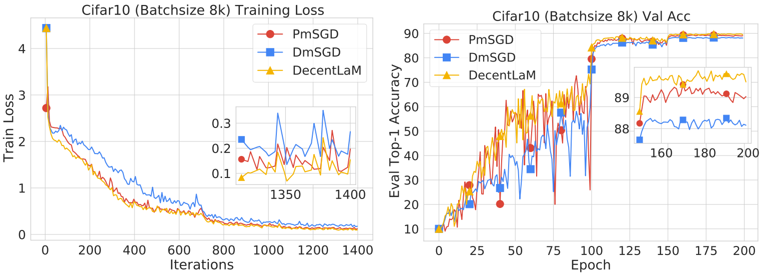


- A numerical verification: full-batch linear regression
- DecentLaM is as fast as DmSGD, and as accurate as DSGD

Go back to the large-batch Cifar-10 Experiment

Experimental setting: CIFAR-10; ResNet-20

Large-batch setting: 8K batch-size per iteration



DecentLaM is much better than DmSGD, and is even better than PmSGD.

Conjecture: For large-batch scenario in which the gradient noise is small, inconsistency bias can help the algorithm to escape the saddle point

Formal convergence theory of DecentLaM

Assumption

(A.1) Each $f_i(x)$ is L-smooth; (A.2) The gradient noise is unbiased and has bounded variance; (A.3) W is positive definite and doubly-stochastic; (A.4) Data heterogeneity is bounded: $\frac{1}{n}\sum_{i=1}^{n}\|\nabla f_i(x) - \nabla f(x)\|^2 \le \hat{b}^2$

Theorem

With appropriate constant learning rate γ (see the paper), DecentLaM will converge at

$$\frac{1}{T}\sum_{k=0}^{T-1}\mathbb{E}\|\frac{1}{n}\sum_{i=1}^{n}\nabla f_i(\bar{x}^{(k)})\|^2$$
$$=O\Big(\underbrace{\frac{1-\beta}{\gamma T}}_{\text{convg. rate}} + \underbrace{\frac{\gamma\sigma^2}{n(1-\beta)} + \frac{\gamma^2\sigma^2}{1-\rho}}_{\text{sto. bias}} + \underbrace{\frac{\gamma^2\hat{b}^2}{(1-\rho)^2}}_{\text{inconsist.bias}}\Big)$$

Formal convergence theory of DecentLaM

- With decaying γ, DecentLaM will converge at rate $O(1/\sqrt{nT})$
- The inconsistency bias of DecentLaM is independent of momentum
- We also establish the convergence rate of DecentLaM with strongly convex cost, see (Yuan et al., 2021)
- W is not necessarily positive-definite in experiments; but it has to be symmetric

Inconsistency bias comparison between various methods

	Strongly-convex	**Non-convex**
DmSGD[3]	N.A.	$O\left(\frac{\gamma^2 M^2}{(1-\beta)^2}\right)$
DmSGD[4]	$O\left(\frac{\gamma^{5/2} M^2}{(1-\beta)^6}\right)$	$O\left(\frac{\gamma^2 M^2}{(1-\beta)^4}\right)$
DmSGD[5]	$O\left(\frac{\gamma^2 b^2}{(1-\beta)^2}\right)$	N.A
DA-DmSGD[6]	N.A.	$O\left(\frac{\gamma^2 \hat{b}^2}{(1-\beta)^2}\right)$
AWC-DmSGD[7]	$O\left(\frac{\gamma^2 M^2}{(1-\beta)^2}\right)$	$O\left(\frac{\gamma^2 M^2}{(1-\beta)^4}\right)$
SlowMo[8]	N.A	N.A
QG-DmSGD[9]	N.A	$O(\gamma^2 \hat{b}^2)$
DecentLaM (Ours)	$\boldsymbol{O(\gamma^2 b^2)}$	$\boldsymbol{O(\gamma^2 \hat{b}^2)}$

[3] (Gao and Huang, 2020)
[4] (Singh et al., 2020)
[5] Derived in this work
[6] (Yu et al., 2019)
[7] (Balu et al., 2020)
[8] (Wang et al., 2019)
[9] (Lin et al., 2021), a concurrent work

Comparison with decentralized primal-dual methods

	Strongly-convex	Non-convex
D2/E-D[10]	0	0
Gradient Tracking[11]	0	0
DecentLaM (Ours)	$O(\gamma^2 b^2)$	$O(\gamma^2 \hat{b}^2)$

- Theoretically, primal-dual methods can completely remove inconsistency bias, which is better than DecentLaM
- Empirically, they are worse than primal methods in validation accuracy
- Conjecture I: no effective acceleration exists for P.-D.
- Conjecture II: some inconsistency bias is beneficial for generalization
- It is still an open question to make decentralized P.-D. useful in DL

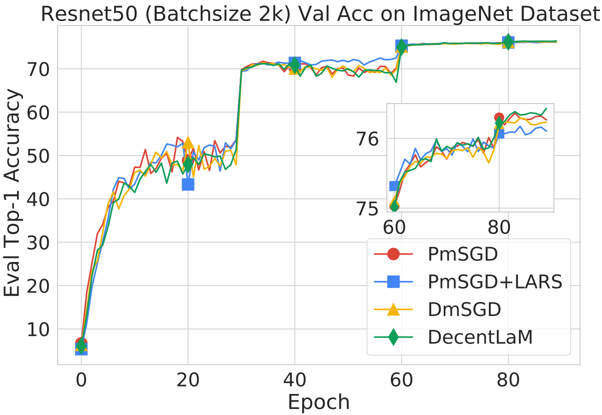
[10] (Yuan et al., 2019; Li et al., 2019; Tang et al., 2018a; Yuan et al., 2020)
[11] (Xu et al., 2015; Di Lorenzo and Scutari, 2016; Nedic et al., 2017; Qu and Li, 2018)

Experiments in Deep Training: Image classification

Image Classification:

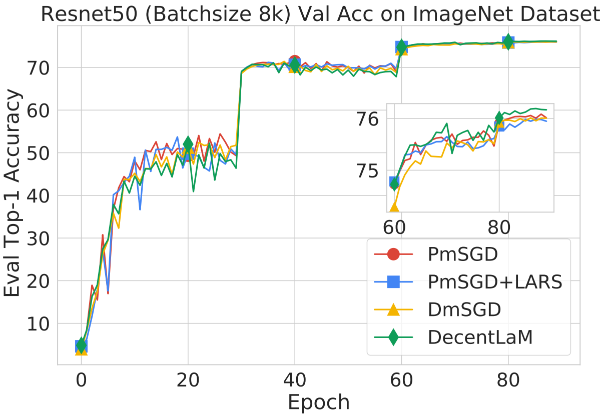
- Model: ResNet-50 ($\sim$25.5M parameters)
- Dataset: ImageNet-1K (1000 classes)
- Size: 1,281,167 training images and 50,000 validation images
- Hardware: 8 GPU $\times$ 8 machines
- We will test the proposed algorithm with batch-size 2K, 8K, 16K, and 32K
- Batchsize $\geq$ 8K is regarded as large batch-size
- Baselines: PmSGD, PmSGD $+$ LARS (layer-wise learning rate), DmSGD

Experiments with batchsize 2K (test accuracy)



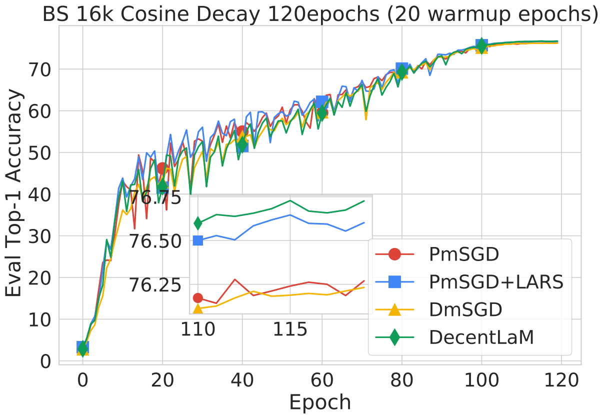
- DecentLaM has similar performance to DmSGD (sto. bias dominates)
- Decentralized methods are no worse than PmSGD

Experiments with batchsize 8K (test accuracy)



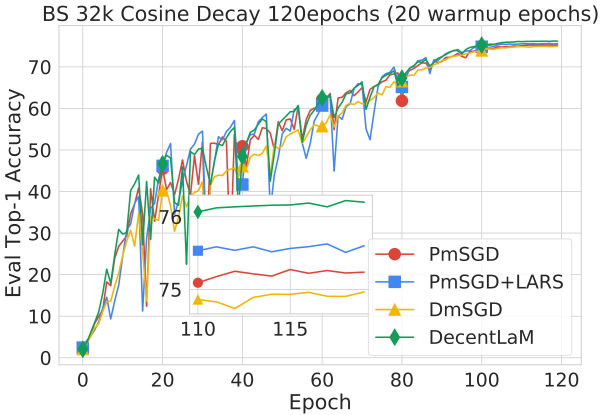
- DecentLaM outperforms DmSGD marginally (sto. bias diminishes)
- DecentLaM also outperforms PmSGD

Experiments with batchsize 16K (test accuracy)



- DecentLaM outperforms DmSGD significantly (incosist. bias diminishes)
- DecentLaM outperforms PmSGD significantly; even better than LARS

Experiments with batchsize 32K (test accuracy)



- DecentLaM outperforms DmSGD significantly (incosist. bias diminishes)
- DecentLaM outperforms PmSGD significantly; even better than LARS

Comparison with more baselines

method	Batch Size			
	2k	8k	16k	32k
PmSGD	76.32	76.08	76.27	75.27
PmSGD+LARS	76.16	75.95	76.65	75.63
DmSGD	76.27	76.01	76.23	74.97
DA-DmSGD	76.35	76.19	76.62	75.51
AWC-DmSGD	76.29	75.96	76.31	75.37
SlowMo	76.30	75.47	75.53	75.33
QG-DmSGD	76.23	75.96	76.60	75.86
D^2-DmSGD	75.44	75.30	76.16	75.44
DecentLaM (Ours)	76.43	76.19	76.73	76.22

Table: Top-1 validation accuracy when training ResNet-50 with different batch sizes.

- DecentLaM can outperform PmSGD (esp. with large-batch)
- D^2-DmSGD is worse than QG-DmSGD and DecentLaM[12]

[12] Similar result was also reported in [Lin et.al., 2021]

Comparison across different DL models

method	ResNet-18	ResNet-34	ResNet-50	MobileNet-v2	EfficientNet
PmSGD	68.3	72.9	76.3	69.5	78.1
DmSGD	68.7	72.4	76.2	72.1	77.5
DecentLaM	70.5	73.4	76.7	72.2	78.3

Table: Top-1 validation accuracy when training ImageNet with $16K$ batchsize.

- DecentLaM outperforms DmSGD with large-batch (as expected)
- DecentLaM also outperforms PmSGD with better generalization error; a surprising result that cannot be explained by current optimization theory
- Conjecture: certain amount of inconsist. bias is beneficial

Running time saving

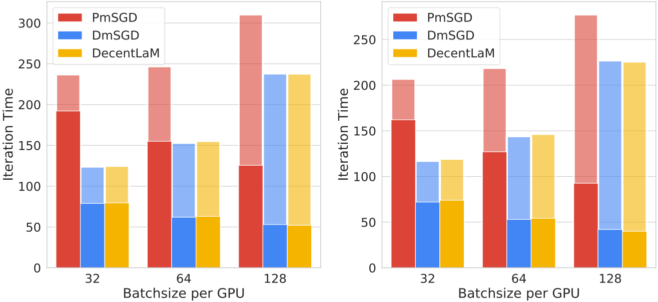


Figure: Runtime comparison on ResNet-50 with different batch sizes and network bandwidth (Left: 10Gbps; Right: 25Gbps). Each column indicates the averaged iteration runtime of 500 iterations. The thick part highlights the comm. overhead.

Experiments in object detection

Dataset	PASCAL VOC		COCO	
Model	R-Net	F-RCNN	R-Net	F-RCNN
PmSGD	79.0	80.3	36.2	36.5
PmSGD+LARS	78.5	79.8	35.7	36.2
DmSGD	79.1	80.5	36.1	36.4
DA-DmSGD	79.0	80.5	36.4	37.0
DecentLaM	79.3	80.7	36.6	37.1

Table: Comparision with different models on PASCAL VOC and COCO datasets. R-Net and F-RCNN refer to RetinaNet and Faster-RCNN respectively.

More results are available in the paper.

Summary

- DmSGD has significant accuracy degradation with large batch-size
- Momentum in DmSGD incurs significant inconsistency bias
- We propose DecentLaM to correct the momentum-incurred bias
- DecentLaM promises both fast and high-quality large-batch training

Part III: Other advanced topics and BlueFog

- Sec. 1 Large-batch deep training
- Sec. 2 An open source decentralized deep training framework: BlueFog

BlueFog: Making Decentralized Algorithms Practical for Optimizaiton and Deep Learning

A library available at `https://github.com/Bluefog-Lib/bluefog`

Aug 5, 2021, Zhejiang University

Main features

- BlueFog is open-source; supports parallel/decentralized methods
- Supports any dynamic and static network topology
- Supports efficient implementation of neighbor-allreduce (partial averaging)
- Suppose both CPU and GPU training through integration with PyTorch
- Wrap up torch optimizers; several codes to run decentralized deep training
- Detailed tutorials with Jupyter notebook on how to use it:

`https://github.com/Bluefog-Lib/bluefog-tutorial`

DNN example

BlueFog has a high-level API that wraps around any torch optimizer.

Example:

```
import torch
import bluefog.torch as bf
bf.init()
...
optimizer = optim.SGD(model.parameters(), lr=lr*bf.size())
optimizer = bf.DistributedNeighborAllreduceOptimzer( \
optimizer, model=model)
...
# Torch training code
```

BlueFog also provides optimizers: Distributed Allreduce, Distributed Hierarchical Neighbor Allreduce, etc.

SPMD (single program, multiple data)

One code for all nodes; different nodes have different data and unique ranks.

```
# hello_world.py
import bluefog.torch as bf
bf.init()
print("I am rank {} in size {}".format(bf.rank(), bf.size()))
```

```
> bfrun -np 2 python hello_world.py
```

```
I am rank 1 in size 2
I am rank 0 in size 2
```

Partial averaging

Example: compute the average of ranks of the nodes

```
import torch
import bluefog.torch as bf
bf.init()

x = torch.Tensor([bf.rank()])

for _ in range(100):
  x = bf.neighbor_allreduce(x)
  print("rank {} has x={}".format(bf.rank(), x))
```

Defaults:

- `bf.init()` creates a static exponential graph
- neighbor-averaging weights are set to $\frac{1}{\text{neighbors}+1}$ for every incoming neighbors and the node itself

```
> bfrun -np 10 python neighbor_avg.py

rank 0 has x=tensor([4.5000])
rank 3 has x=tensor([4.5000])
rank 9 has x=tensor([4.5000])
rank 1 has x=tensor([4.5000])
rank 7 has x=tensor([4.5000])
rank 4 has x=tensor([4.5000])
rank 2 has x=tensor([4.5000])
rank 6 has x=tensor([4.5000])
rank 5 has x=tensor([4.5000])
rank 6 has x=tensor([4.5000])
```

Partial averaging using dynamic subgraphs

Example: Default one-peer exponential averaging

```
dynamic_neighbors = topology_util.GetDynamicSendRecvRanks(
bf.load_topology(), bf.rank())

for _ in range(maxite):
  to_neighbors, from_neighbors = next(dynamic_neighbors)

  avg_weight = 1/(len(from_neighbors) + 1)

  xi = bf.neighbor_allreduce(xi, name='x',
  self_weight=avg_weight,
  neighbor_weights={r: avg_weight for r in from_neighbors},
  send_neighbors=to_neighbors)
```

You can replace `GetDynamicSendRecvRanks()` with your own.

Decentralized gradient descent (Nedic and Ozdaglar, 2009)

To approximate solve

$$\underset{\mathbf{x}}{\text{minimize}} \;\; \alpha \sum_{i=1}^{n} f_i(x_i) \qquad \text{subject to } x_1 = \cdots = x_n,$$

we can apply *decentralized gradient descent*:

$$\mathbf{x}^{k+1} = W\mathbf{x}^k - \alpha \nabla f(\mathbf{x}^k).$$

Implementation using static exp2:

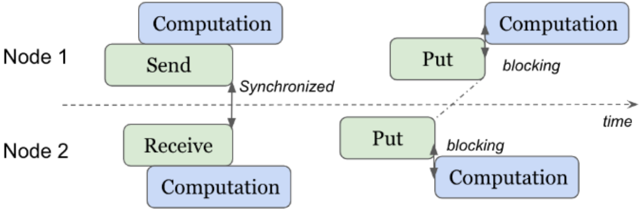
```
# DGD recursion
for k in range(maxite):
  xi = bf.neighbor_allreduce(xi) - alpha*ComputeGrad(fi,xi)
```

Blocking and asynchrony

Each node has two threads: communication thread and computation thread

- **non-blocking:** allow concurrent threads to save time
- blocking: computation starts after communication completes

Synchronization is similar concept but applies to operations across different nodes. All collective communications are synchronous.



Left: nonblocking but synchronized; **Right:** blocking, may or may not sync'd

By default, BlueFog is blocking and synchronized, but it also supports non-blocking and asynchronous operations

To save time, we ask neighbor allreduce $W\mathbf{x}^k$ not to block computation $\nabla f(\mathbf{x}^k)$, so they can run concurrently.

```
1 for k in range(maxite):
2   handle = bf.neighbor_allreduce_nonblocking(xi)
3   gradi = ComputeGrad(fi, xi)
4   avg_x = bf.wait(handle)
5   xi = avg_x - alpha*gradi
```

Since Line 5 must wait for the result of $W\mathbf{x}^k$.

EXTRA (Shi et al., 2015)

EXTRA was the first method that solves

$$\underset{x}{\text{minimize}} \sum_{i=1}^{n} f_i(x_i) \qquad \text{subject to } x_1 = \cdots = x_n$$

with a constant α. One form of this method is

$$\begin{cases} \mathbf{x}^1 = W\mathbf{x}^0 - \alpha \nabla f(\mathbf{x}^0), \\ \mathbf{x}^{k+1} = W(2\mathbf{x}^k - \mathbf{x}^{k-1}) - \alpha(\nabla f(\mathbf{x}^k) - \nabla f(\mathbf{x}^{k-1})), \quad k = 1, 2, \cdots \end{cases}$$

The code structure is similar to DGD. Non-blocking communication can accelerate the code.

Gradient-Tracking

DIGing Nedic et al. (2017) is a tracking-based method. For static W, DIGing is a special case of EXTRA. However, DIGing works for dynamic W.

$$\begin{cases} \mathbf{x}^{k+1} = W^{(k)}\mathbf{x}^k - \alpha \mathbf{y}^k \\ \mathbf{y}^{k+1} = W^{(k)}\mathbf{y}^k + \nabla f(\mathbf{x}^{k+1}) - \nabla f(\mathbf{x}^k) \end{cases}$$

$(\mathbf{y}^k)_k$ a tracking sequence converging to $\lim_k \frac{1}{n}\sum_{i=1}^n \nabla f_i(\mathbf{x}^k)$ if it exists.

```
xi = np.zeros((d,1))
yi = fi_grad_prev = ComputeGrad(fi, xi)
for k in range(maxite):
  self_weight, recv_weights = ComputeWeights(k, bf.rank())
  xi = bf.neighbor_allreduce(xi, self_weight, recv_weights) \
  - alpha*yi
  gi = ComputeGrad(fi, xi)
  yi = bf.neighbor_allreduce(gi, self_weight, recv_weights) \
  + gi - gi_prev
  gi_prev = gi.copy()
```

Summary

- Decentralized computing can accelerate large-scale deep training
- Exponential graphs are provably efficient for decentralized deep training
- Periodic global averaging can further accelerate decentralized deep training
- We develop a GitHub ropo to help implement decentralized training easily

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